The Open University



# MST121

# Assignment Booklet II 2008B

**Contents** **Cut-off date**



This booklet contains the final two assignments for the course: TMA 04 and CMA 42. These assignments cover the whole course, and should help you consolidate your work on it.

The work you submit must be your own and not copied from, or provided by, others. There is further information on plagiarism in the *Assessment Handbook* (accessible via your StudentHome web page).

**Frequently-asked questions about TMA 04 and CMA 42**

**Can I pass the course without submitting TMA 04?**
No. To pass the course you must achieve *both* of the following:

- an overall assessment score of at least 40%;
- at least 30% on TMA 04.

Details of how your overall assessment score is calculated are given in the *Course Guide* and on the *Study Calendar*. The Assessment Calculator, accessible via your StudentHome web page, may also be helpful.

**Can I have an extension for TMA 04 or CMA 42?**
No. Extensions are not permitted for this final TMA, nor for any CMA. If you have *serious* problems in meeting the deadlines for TMA 04 and CMA 42, then you may be allowed to defer them to the next presentation of the course. See the information on pages 17–19.

**Can my scores for TMA 04 or CMA 42 be substituted?**
No. (Substitution is explained in the *Assessment Handbook*.)

*[Continued overleaf]*

**When should I start work on TMA 04 and CMA 42?**
By two to three weeks before the respective cut-off dates, at the latest. You will probably find that you need to revise parts of the course, so leave yourself plenty of time for this.

**What if I am having problems finding enough time?**
You should start work on TMA 04 and CMA 42 no later than two to three weeks before their respective cut-off dates, even if you have not finished working through the course materials. You should do as much as you can on TMA 04 and CMA 42, giving priority to TMA 04, and submit what you have done by the cut-off dates. If your shortage of time is due to adverse circumstances, then see also the information on pages 17–19.

**How should I submit TMA 04?**
Send it to your tutor, together with a TMA form (PT3), to arrive by the cut-off date. Make sure that you allow sufficient time in the post, and *check that the postage is sufficient*. You are advised to keep a copy of it in case of loss in the mail, and to obtain proof of posting, because if it arrives after the cut-off date then you will be asked to provide proof that you posted it in time. Remember to write your Personal Identifier and the TMA number in the boxes provided on the TMA form, and to sign the plagiarism statement at the top.

**How should I submit CMA 42?**
Either online, from your StudentHome web page, up to the cut-off date, or by sending the completed paper CMA form to arrive at Walton Hall by the cut-off date.

**What will happen if my TMA 04 or CMA 42 is received after the cut-off date?**
It will not be accepted for credit (and therefore, in the case of TMA 04, you will fail the course) unless you are able to prove that it was posted in sufficient time to arrive before the cut-off date.

**When will I receive my course result, and when will I receive feedback on TMA 04 and CMA 42?**
You should receive your result by mid-December. It will be posted to you, and will also be available via your StudentHome web page. You will receive feedback on TMA 04, and the answers to CMA 42, after your course result has been issued.

**If I do not achieve the criteria for a pass result, will I be allowed to attempt the assessment again?**
In most cases, no: you will receive a fail result, and you will not be allowed to attempt the assessment again, except by re-taking the course. However, if your overall weighted average is over, or close to, 40%, and the Examination and Assessment Board considers that you have made a reasonable attempt at TMA 04, then it may decide to offer you the opportunity to attempt the new versions of TMA 04 and CMA 42 in the next presentation of the course. In this case your course result letter will state 'pending' instead of pass or fail, and say that 'The Examination and Assessment Board wishes to offer you the opportunity to resubmit one or more elements of your assessment. Details are being sent to you separately.' You should receive the new assignments, together with instructions for submitting them, within two weeks of receiving your course result letter. The cut-off dates are in early June. If you wish to accept the offer, your final course result will be based on your scores for these two assignments and on the scores you have already obtained for the assignments in *Assignment Booklet I*.

**Points to note when preparing solutions to TMA questions**

- Contact your tutor if the meaning of any part of a question does not seem clear.
- Your solutions should not involve the use of Mathcad, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software, except OUStats.
- Except where you are asked simply to 'write down' or 'state' an answer, justify your answers by showing your working and explaining your reasoning. The solutions to the Activities and Exercises in the Chapters and Exercise Booklets will give you an idea of how much detail is needed. You may not receive full marks for a correct final answer that is not supported by working. You may receive some marks for working even if your final answer is incorrect or your solution is incomplete.
- Whenever you perform a calculation using a numerical answer found earlier, you should use the full-calculator-accuracy version of the earlier answer to avoid rounding errors.
- Number all of your pages, including any computer printouts.
- Indicate in each solution the page numbers of any computer printouts associated with that solution.
- Write your name and personal identifier on each page.
- Use dark ink, as your answers may be photocopied.
- The marks allocated to the parts of the questions are indicated in brackets in the margin. Each TMA is marked out of 100. Your overall score for a TMA will be the sum of your marks for each question part.
- If possible, check the 'News' section of the course website immediately before beginning each assignment, in case any late information, such as errata, has been posted. If you later suspect that an assignment question contains an error or is unsound, check the course website for errata. If you find nothing helpful there, or if you are unable to access the website, contact your tutor about the problem.

This assignment covers the whole course.

You should answer **ALL FOUR** questions. One of the requirements for passing the course is that you must obtain a score of at least 30 out of 100 for this assignment.

**Question 1** – 25 marks

In this question, positions are given with reference to a Cartesian coordinate system whose $x$- and $y$-axes point due East and due North, respectively. Distance is measured in kilometres.

An aeroplane flies in a straight line from city A, at $(300, -200)$, to city B, at $(-100, 600)$.

(a) (i) Find the equation of the line of flight of the aeroplane. [3]

(ii) Find the direction of travel of the aeroplane, as a bearing, with the angle in degrees correct to one decimal place. [3]

(iii) Find the distance between cities A and B, to the nearest kilometre. [2]

(b) After landing at city B, the aeroplane flies in a straight line in the direction S 23°W, to city C, before finally flying in a straight line in the direction N 54°E back to city A.

(i) Draw a diagram, in the form of a triangle, showing the three flights of the aeroplane. Calculate and mark the three angles on your diagram, in degrees to one decimal place. [4]

(ii) Find the distance between cities A and C, to the nearest kilometre. [3]

(c) This part of the question is about the aeroplane's flight from city A to city B. During this flight, it flies within range of an air traffic control centre at position $(300, 0)$.

(i) Find parametric equations for the line of flight of the aeroplane. Your equations should be in terms of the parameter $t$, and should be such that the aeroplane is at city A when $t = 0$ and at city B when $t = 1$. [2]

(ii) Write down an expression, in terms of $t$, for the square of the distance between the air traffic control centre and the aeroplane at the point with parameter $t$ on the line of flight of the aeroplane. Simplify your answer.

(The distance required here, and in part (c)(iii), is the 'horizontal distance', that is, the distance between the air traffic control centre and the point on the ground immediately below the aeroplane.) [3]

(iii) Use your answer to part (c)(ii), and the method of completing the square, to determine the distance, to the nearest kilometre, between the air traffic control centre and the aeroplane at the point on the line of flight of the aeroplane where it is closest to the air traffic control centre. [5]

**Question 2** – 25 marks

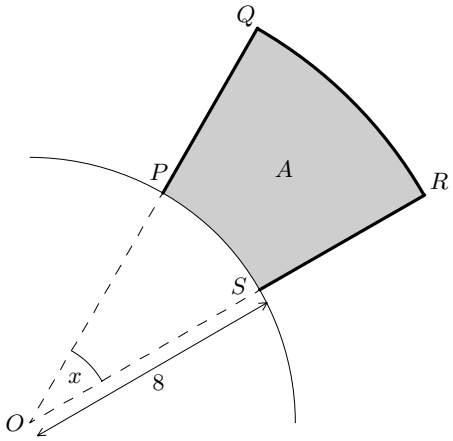
*This question requires the use of Mathcad throughout. For each of parts (a)–(c) you should provide an appropriate printout, though a printout on one page may cover your answers to several parts. Annotate your printouts with Mathcad text or handwriting, or reference them from a separate sheet, in order to explain clearly what you have done and what your conclusions are.*

A flexible wire $PQRS$, of total length 20 metres, is bent into a three-edged planar shape, and its ends $P$, $S$ are placed against a disc of radius 8 metres with centre $O$, as shown in the diagram below. (The arc $PS$ is not part of the wire.) The end-segments $PQ$ and $RS$ of the wire lie along straight lines through $O$, while the arc $QR$ forms part of a circle with centre $O$ and subtends an angle $x$ (in radians) at $O$.

This question concerns the area $A$ enclosed between the wire and the edge of the disc, which is shown shaded below. This area can be expressed by $A = f(x)$, where

$$f(x) = \frac{8x(5-2x)(13+2x)}{(2+x)^2} \quad (0 \le x \le 2.5).$$

(*You are NOT asked to show this.*)




*In part (a), you may either use the Mathcad graph plotter file (121A3-04) or plot the graph in a new worksheet of your own.*

(a) (i) Plot the graph of the function $f(x)$. Your graph should cover the interval $[0, 2.5]$ in the $x$-direction and $[0, 50]$ in the $y$-direction. [*4*]

(ii) By using the 'Trace' facility (and also, if you wish, the 'Zoom' facility), estimate to two decimal places the coordinates of the point on this graph at which $y = f(x)$ takes its maximum value. [*2*]

(iii) On the same graph, plot the line $y = 25$. Using the 'Trace' facility, estimate to two decimal places both solutions of the equation $f(x) = 25$. (These solutions give the values of $x$ for which the shaded area is $25\,\mathrm{m}^2$.) [*3*]

(b) Each of the following recurrence relations has the property that if a sequence generated by the recurrence relation converges to a limit in the interval $[0, 2.5]$, then that limit must be a solution of the equation $f(x) = 25$. (*You are NOT asked to show this.*)

$$\mathbf{A}: \quad x_{n+1} = \frac{25(2 + x_n)^2}{8(5 - 2x_n)(13 + 2x_n)} \quad (n = 0, 1, 2, \ldots)$$

$$\mathbf{B}: \quad x_{n+1} = \frac{10}{21 - 4\sqrt{2(9 - x_n)}} \quad (n = 0, 1, 2, \ldots)$$

(i) For each of these recurrence relations, generate the sequence with starting value $x_0 = 1.75$, and tabulate your results to six decimal places. Which sequence converges more rapidly? (That is, which sequence gives an estimate with specified accuracy for its limit with the smaller value of $n$?) [*5*]

(ii) Use your tabulated results to write down the solutions of the equation $f(x) = 25$ to six decimal places. [*3*]

(c) This part of the question concerns finding the maximum value of the function $f(x)$, as estimated in part (a)(ii), and hence the maximum possible value of the shaded area $A$.

*You may need to put $x := x$ in your worksheet before answering part (c)(i).*

(i) Use symbolic differentiation and the 'simplify' keyword to find an expression for the derivative $f'(x)$. [*3*]

(ii) The maximum value of $f(x)$ occurs where $f'(x) = 0$. Use a solve block to solve the equation $f'(x) = 0$ for $x$, giving your answer to six decimal places. [*3*]

(iii) Calculate the corresponding maximum possible value of the area $A$, giving your answer to four decimal places. [*2*]

**Question 3** – 25 marks

(a) (i) Use the Composite Rule to differentiate the function

$$f(x) = \sqrt{10 + x^2}.$$ [5]

(ii) Use the Quotient Rule and your answer to part (a)(i) to show that the function

$$g(x) = \frac{e^{x/7}}{\sqrt{10 + x^2}}$$

has derivative

$$g'(x) = \frac{e^{x/7}(x^2 - 7x + 10)}{7(10 + x^2)^{3/2}}.$$ [4]

(iii) Find any stationary points of the function $g(x)$ defined in part (a)(ii), and use the First Derivative Test to classify each stationary point as a local maximum or a local minimum of $g(x)$. [5]

(iv) Using your answers to parts (a)(ii) and (a)(iii), find the area bounded by the graph of

$$y = \frac{100e^{x/7}(5 - x)(x - 2)}{(10 + x^2)^{3/2}}$$

and by the $x$-axis. Give your answer to four significant figures. [4]

(b) (i) Using your answer to part (a)(i), find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{10 + y^2}}{y} \quad (x > 0,\ y > 0),$$

giving the solution in implicit form. [4]

(ii) Find the particular solution of the differential equation in part (b)(i) for which $y = \sqrt{6}$ when $x = 0$, and then give this particular solution in explicit form. [3]

**Question 4** – 25 marks

*This question requires the use of OUStats. For each of parts (a)(iii) and (b)(i) you should provide an appropriate printout.*

(a) The file AGES.OUS contains data on the ages in months of a sample of 113 students enrolled on an elementary statistics course.

It is required to obtain a histogram of the data with the bars corresponding to years: for example, one bar is to represent all ages recorded as 240 months (20 years) or more but less than 252 months (21 years). The smallest age is to be represented in the first bar.

(i) State the interval width that should be used for the histogram. [*1*]

(ii) Find the smallest age in the sample. Assuming that the ages are rounded down to the nearest month (for example, 254 months and 27 days would be recorded as 254 months), explain why the most appropriate first interval starting value for the histogram is exactly 204. [*3*]

(iii) Obtain a histogram of the data, using the interval width and first interval starting value from parts (a)(i) and (a)(ii). State a reason why a normal distribution might not be a suitable model for the variation in the data. [*4*]

(iv) On a separate diagram, but using a similar scale, sketch a curve that you think might be a suitable model for the variation in the ages of students enrolled on similar elementary statistics courses. [*3*]

(v) Explain how you might use your model to estimate the proportion of students enrolled on similar elementary statistics courses who are aged less than 300 months. [*2*]

(b) The file BEARS.OUS contains the results of aerial surveys carried out on each of 20 days in a particular part of Alaska. The column headed 'Windspeed' contains the average wind speed on each day, and the column headed 'Bears' contains the corresponding number of black bears sighted. The source does not report the units in which the wind speed was measured.

You are asked to consider the relationship between the average wind speed and the number of black bears sighted.

(i) Obtain a scatterplot of Bears against Windspeed, using Windspeed as the explanatory variable, and add the least squares fit line to the plot. [*3*]

(ii) Obtain the equation of the least squares fit line of Bears on Windspeed. If you use labels other than Bears and Windspeed for the variables, specify which variable represents Bears and which represents Windspeed. [*2*]

(iii) Use the scatterplot to identify the three points that have the greatest residuals, and write down the data pairs corresponding to these points. Comment on what these points seem to indicate about the model obtained in part (b)(ii). [*4*]

(iv) Using the model obtained in part (b)(ii), find the predicted number of black bears that would be sighted on a day with average windspeed 40.0. Comment on whether you would expect this prediction to be accurate, and why. [*3*]

This CMA contains 28 questions.

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*Questions 1 to 7 are on Block A.*

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**Question 1**

Choose the TWO options that are **false** statements.

*Options*

**A** $(\sqrt{5})^6 = 125$

**B** $\dfrac{1}{5^{3/4}} = 5^{4/3}$

**C** $\left(\frac{1}{32}\right)^{-2/5} = 4$

**D** $\dfrac{a^3}{a^4 \times a^5} = a^{1/3} \quad (a \text{ in } \mathbb{R},\ a > 0,\ a \neq 1)$

**E** $\sqrt{26}$ cannot be expressed as a recurring decimal.

**F** The decimal representation of $\sqrt{26}$, correct to three significant figures, is 5.10.

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**Question 2**

A sequence is defined by

$$u_n = 12 \times (-0.45)^n + 1.9 \quad (n = 1, 2, 3, \ldots).$$

Choose the option that describes the long-term behaviour of the sequence.

*Options*

**A** $u_n$ becomes arbitrarily small (that is, $u_n \to 0$ as $n \to \infty$).

**B** $u_n$ becomes arbitrarily large and negative (that is, $u_n \to -\infty$ as $n \to \infty$).

**C** $u_n$ becomes arbitrarily large and positive (that is, $u_n \to \infty$ as $n \to \infty$).

**D** $u_n$ is unbounded and alternates in sign.

**E** $u_n$ approaches 1.9 (that is, $u_n \to 1.9$ as $n \to \infty$).

**F** $u_n$ approaches 13.9 (that is, $u_n \to 13.9$ as $n \to \infty$).

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**Question 3**

Choose the option that represents the equation of the line perpendicular to the line $9x - 4y = 17$, passing through the point $(1, -2)$.

*Options*

**A** $9x + 4y = -1$ **B** $4x + 9y = -14$ **C** $4x - 9y = -22$

**D** $9x + 4y = 1$ **E** $4x + 9y = 14$ **F** $4x - 9y = 22$

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**Question 4**

The method of completing the square can be used to write the expression $5x^2 + 20x - 12$ in the form $a(x + b)^2 + c$, where $a$, $b$ and $c$ are constants. Choose the option that gives the value of $c$.

*Options*

**A** $-32$ **B** $-16$ **C** $-12$ **D** $-8$ **E** $4$ **F** $8$

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**Question 5**

Choose the option that gives the domain of the function

$$f(x) = \frac{1}{\sqrt{(6-x)(5+x)}}.$$

*Options*

**A** $\mathbb{R}$ **B** $(-5, 6)$ **C** $[-5, \infty)$

**D** $(-\infty, 6]$ **E** $(5, 6)$ **F** $[5, 6]$

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**Question 6**

Choose the option that is a **true** statement.

*Options*

**A** $(\arccos x) \times (\cos x) = 1$, for all $x$ in $[-1, 1]$

**B** $\sin(\arcsin \pi) = \pi$

**C** $\dfrac{\arcsin x}{\arctan x} = \arccos x$, for all $x$ in $(0, 1)$

**D** $\arcsin(\sin \pi) = \pi$

**E** $\arctan(\frac{1}{6}\pi) = \frac{1}{3}\sqrt{3}$

**F** $\arccos(\tan(\frac{3}{4}\pi)) = \pi$

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**Question 7**

Choose the TWO options that are **false** statements.

*Options*

**A** $3\ln(x-2) = \dfrac{x^3}{8} \quad (x > 2)$

**B** $\log_{10}\left(\dfrac{100}{10^{2x}}\right) = 2 - 2x \quad (x \text{ in } \mathbb{R})$

**C** $\ln\left(e^{2-2x}\right) = 2 - 2x \quad (x \text{ in } \mathbb{R})$

**D** $\ln(5e^{2x}) = 5 + 2x \quad (x \text{ in } \mathbb{R})$

**E** $\ln\left(\dfrac{5x^2}{x-1}\right) = \ln 5 + 2\ln x - \ln(x-1) \quad (x > 1)$

**F** $2^{10\log_2 x} = x^{10} \quad (x > 0)$

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*Questions 8 to 14 are on Block B.*

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**Question 8**

Choose the option that gives the value of $\sum_{i=78}^{109}(137+5i)$.

*Options*

**A** 5184 **B** 7376 **C** 13 944 **D** 17 920

**E** 19 207 **F** 19 344 **G** 27 178 **H** 34 359

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**Question 9**

Choose the TWO options that give the term $P_1$, and the long-term behaviour, of the sequence generated by the recurrence system

$$P_0 = 2400, \quad P_{n+1} - P_n = 0.45P_n\left(1 - \frac{P_n}{16\,000}\right) \quad (n = 0, 1, 2, \ldots).$$

*Options*

**A** 3165 **B** 3318 **C** 3642 **D** 4440

**E** The sequence converges to 16 000, with values always below 16 000.

**F** The sequence converges to 16 000, with values eventually alternating above and below 16 000.

**G** The sequence tends to a 2-cycle.

**H** The sequence shows chaotic variation.

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**Question 10**

Choose the THREE options that define sequences that do not converge.

*Options*

**A** $P_0 = 50, \quad P_{n+1} - P_n = 0.9P_n\left(1 - \frac{P_n}{450}\right) \quad (n = 0, 1, 2, \ldots)$

**B** $P_0 = 100, \quad P_{n+1} - P_n = 2.4P_n\left(1 - \frac{P_n}{400}\right) \quad (n = 0, 1, 2, \ldots)$

**C** $P_0 = 250, \quad P_{n+1} - P_n = 2.8P_n\left(1 - \frac{P_n}{650}\right) \quad (n = 0, 1, 2, \ldots)$

**D** $a_n = \frac{5n-3}{1+4n} \quad (n = 0, 1, 2, \ldots)$

**E** $a_n = \frac{70}{3(0.3)^n} \quad (n = 0, 1, 2, \ldots)$

**F** $a_n = \frac{7n^4 - 14n^2}{1+n^5} \quad (n = 0, 1, 2, \ldots)$

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**Question 11**

The variation in a population is modelled by the recurrence relation

$$\begin{pmatrix} J_{n+1} \\ W_{n+1} \\ E_{n+1} \end{pmatrix} = \begin{pmatrix} 0.897 & 0.056 & 0 \\ 0.065 & 0.958 & 0 \\ 0 & 0.017 & 0.910 \end{pmatrix} \begin{pmatrix} J_n \\ W_n \\ E_n \end{pmatrix},$$

where $J_n$, $W_n$ and $E_n$ are the sizes, in millions, of the subpopulations of juveniles, workers and elderly, respectively, $n$ years after 1 January 2008. On that date there were 10.54 million juveniles, 21.34 million workers and 7.81 million elderly. Choose the option that gives the number of workers, in millions, to two decimal places, predicted by the model for 1 January 2009.

*Options*

**A** 7.47 **B** 10.65 **C** 20.14 **D** 20.44

**E** 21.13 **F** 21.17 **G** 21.87 **H** 39.25

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**Question 12**

Three forces (and no others) act upon an object, which remains at rest. The forces are represented by the vectors **P**, **Q** and **R**, where

$$\mathbf{P} = -5\mathbf{i} - 3\mathbf{j} \quad \text{and} \quad \mathbf{Q} = 4\mathbf{i} + 9\mathbf{j}.$$

Choose the TWO options that give the magnitude and direction of the vector **R**, to one decimal place.

*Options*

**A** Magnitude 6.1 **B** Magnitude 10.8 **C** Direction $-99.5°$

**D** Direction $-80.5°$ **E** Direction $-33.7°$ **F** Direction $33.7°$

**G** Direction $80.5°$ **H** Direction $99.5°$

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**Question 13**

The vector **a** has magnitude 3.4 and direction $-113°$, and the vector **b** has magnitude 4.7 and direction $19°$. Choose the option that gives the component form of the vector $3\mathbf{a} - \mathbf{b}$, to one decimal place.

*Options*

**A** $-8.4\mathbf{i} - 10.9\mathbf{j}$ **B** $-8.4\mathbf{i} - 7.8\mathbf{j}$ **C** $-5.7\mathbf{i} - 7.8\mathbf{j}$

**D** $-5.7\mathbf{i} - 4.7\mathbf{j}$ **E** $0.5\mathbf{i} - 5.4\mathbf{j}$ **F** $0.5\mathbf{i} + 3.2\mathbf{j}$

**G** $6.3\mathbf{i} - 5.4\mathbf{j}$ **H** $6.3\mathbf{i} + 4.7\mathbf{j}$

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**Question 14**

A triangle has side lengths 7, 8 and 9. Choose the option that gives its largest angle, to one decimal place.

*Options*

**A** $58.4°$ **B** $65.2°$ **C** $71.6°$ **D** $73.4°$

**E** $78.6°$ **F** $82.5°$ **G** $86.4°$ **H** $121.6°$

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*Questions 15 to 21 are on Block C.*

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**Questions 15 and 16**

The function $f$ is given by

$$f(x) = \ln(\cos x - \sin x) \quad (-\tfrac{3}{4}\pi < x < \tfrac{1}{4}\pi).$$

By applying in turn the Composite and Quotient Rules, choose the option that gives an expression for each of the following.

**15** $f'(x)$

**16** $f''(x)$

*Options for Questions 15 and 16*

**A** $\dfrac{1}{\cos x - \sin x}$ **B** $\dfrac{\sin x + \cos x}{\sin x - \cos x}$ **C** $\dfrac{\sin x - \cos x}{\sin x + \cos x}$

**D** $\dfrac{1}{\cos^2 x - \sin^2 x}$ **E** $-\dfrac{1}{\cos x + \sin x}$ **F** $-\dfrac{2}{(\sin x - \cos x)^2}$

**G** $\dfrac{\sin x - \cos x}{(\sin x + \cos x)^2}$ **H** $\dfrac{\cos x \sin x}{(\sin x - \cos x)^2}$

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**Question 17**

By first applying the Product Rule, choose the option that gives (correct to two decimal places) the overall maximum value of the function

$$f(x) = 25e^{-3x}\sin(2x)$$

on the interval $[0, \pi]$.

*Options*

**A** 0.51 **B** 2.37 **C** 5.74 **D** 8.01

**E** 9.79 **F** 12.46 **G** 14.63 **H** 17.20

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**Question 18**

By using *either* equation (2.3) *or* equation (2.4) in Chapter C2, choose the option that gives an expression for the indefinite integral

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx \quad (x > 0).$$

In each option, $c$ is an arbitrary constant.

*Options*

**A** $\ln\left(\sqrt{x}\left(1+\sqrt{x}\right)\right) + c$ **B** $2\ln\left(1+\sqrt{x}\right) + c$

**C** $\ln\left(\dfrac{\sqrt{x}}{1+\sqrt{x}}\right) + c$ **D** $2\ln\left(\dfrac{1+\sqrt{x}}{\sqrt{x}}\right) + c$

**E** $-\dfrac{2}{1+\sqrt{x}} + c$ **F** $\dfrac{1}{\sqrt{x}\,(1+\sqrt{x})} + c$

**G** $-\dfrac{2\sqrt{x}}{1+\sqrt{x}} + c$ **H** $\dfrac{\sqrt{x}}{2\,(1+\sqrt{x})^3} + c$

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**Question 19**

Choose the option that gives (correct to two decimal places) the area of the region which lies between the lines $x = \frac{1}{6}\pi$ and $x = \frac{5}{6}\pi$, above the $x$-axis and below the graph of

$$y = \sin x\,(2 + \sin x).$$

*Options*

**A** 1.08 **B** 1.75 **C** 2.58 **D** 3.20

**E** 3.89 **F** 4.31 **G** 4.94 **H** 5.67

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**Question 20**

Choose the option that gives the solution of the initial-value problem

$$\frac{dy}{dx} = \tfrac{1}{2}e^{-x/4}y\sqrt{y} \quad (y > 0), \quad y = \tfrac{1}{4} \text{ when } x = 0.$$

*Options*

**A** $y = \dfrac{1}{(1 + e^{-x/4})^2}$ **B** $y = \dfrac{1}{\sqrt{15 + e^{-x/4}}}$

**C** $y = \dfrac{1}{\sqrt{17 - e^{-x/4}}}$ **D** $y = 2e^{-x/4} - \frac{7}{4}$

**E** $y = \frac{1}{8}\sqrt{e^{-x/4} + 3}$ **F** $y = \frac{1}{16}(3 - e^{-x/4})^2$

**G** $y = \dfrac{1}{\sqrt{8(1 + e^{-x/4})}}$ **H** $y = \dfrac{1}{4(2 - e^{-x/4})^2}$

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**Question 21**

The mass $m$ of a radioactive substance present at time $t$ can be modelled by the equation $m = m_0e^{-kt}$, where $m_0$ is the initial mass of the substance and $k$ is a constant. After 30 years, one third of the substance has decayed (and therefore two thirds remain). Choose the option that gives (in years, to the nearest year) the half-life of the substance.

*Options*

**A** 39 **B** 43 **C** 47 **D** 51

**E** 55 **F** 59 **G** 63 **H** 67

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*Questions 22 to 28 are on Block D.*

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**Question 22**

Two dice each have faces numbered 1 to 6. They are rolled together. Choose the option that is the probability that the sum of the numbers obtained is 7.

*Options*

**A** $\frac{5}{36}$ **B** $\frac{1}{6}$ **C** $\frac{7}{36}$ **D** $\frac{1}{3}$ **E** $\frac{1}{2}$ **F** $\frac{7}{12}$

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**Question 23**

An urn contains three black balls and two gold balls. A ball is drawn at random and returned to the urn. A second ball is then drawn at random. Choose the option that gives the probability that at least one of the balls drawn was gold.

*Options*

**A** $\frac{1}{25}$ **B** $\frac{4}{25}$ **C** $\frac{6}{25}$ **D** $\frac{9}{25}$ **E** $\frac{16}{25}$ **F** $\frac{7}{10}$

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**Questions 24 and 25**

The distribution of the weights of the contents of packs of tea bags, labelled as containing 125 g, has mean 125.2 g and standard deviation 1.5 g.

**24** Choose the option that is closest to the standard error of the mean weight (in grams) of the contents for samples of 50 packs of tea bags.

*Options for Question 24*

**A** 0.030 **B** 0.212 **C** 0.300

**D** 1.500 **E** 2.504 **F** 17.706

*You will need to use OUStats to answer Question 25.*

**25** The mean weight of the contents for samples of 28 packs of the same tea bags has standard error 0.283 g. Choose the option that is closest to the probability that the mean weight of the contents for a sample of 28 packs of tea bags is between 124.8 g and 125.2 g.

*Options for Question 25*

**A** 0.009 **B** 0.063 **C** 0.105

**D** 0.140 **E** 0.421 **F** 0.470

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**Question 26**

The US Environmental Protection Agency is particularly concerned when the ozone concentration in air exceeds 80 parts per billion (ppb). Suppose that a sample of 34 measures of ozone concentration, from counties across the Midwest, has an average of 75 ppb with standard deviation 6 ppb. Choose the option that gives an approximate 95% confidence interval (in ppb, to two decimal places) for the mean ozone concentration in the Midwest.

*Options*

**A** $(63.24, 86.76)$ **B** $(65.13, 84.87)$ **C** $(68.24, 91.76)$

**D** $(72.98, 77.02)$ **E** $(73.31, 76.69)$ **F** $(77.98, 82.02)$

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**Question 27**

A market research company asked 100 citizens of one country and 150 citizens of a second country the number of times per year that they visit the cinema. For the citizens of the first country, the mean number of visits per year was 3.4 with standard deviation 1.5. For the citizens of the second country, the mean number of visits per year was 2.1 with standard deviation 1.7. Choose the option that is closest to the estimated standard error (ESE) of the difference between the two sample mean numbers of times per year that the citizens of the two countries visit the cinema.

*Options*

**A** 0.026 **B** 0.057 **C** 0.162

**D** 0.204 **E** 0.219 **F** 0.381

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**Question 28**

A study was carried out to investigate the effect of position in a garden on the heights of sunflowers. The heights of 100 sunflowers were measured. Fifty-five of the sunflowers were grown in a shady part of the garden and the rest were grown in a sunny part. The mean height of the sunflowers grown in the shady part was 2.85 m, with standard deviation 0.495 m, and the mean height of the sunflowers grown in the sunny part was 3.23 m, with standard deviation 0.662 m. The estimated standard error of the difference between the two sample mean sunflower heights is 0.119 m.

The two-sample $z$-test is to be used to determine whether there is a difference, at the 5% significance level, between the heights for sunflowers grown in the shady and sunny parts of the garden. Choose the TWO options that give the magnitude of the test statistic $z$, to two decimal places, and the conclusion obtained from the test.

*Options*

**A** 1.40 **B** 1.79 **C** 1.86

**D** 3.19 **E** 9.17 **F** 11.85

**G** There seems to be a significant difference between the heights of sunflowers grown in the shady and sunny parts of the garden, at the 5% significance level.

**H** There seems to be no significant difference between the heights of sunflowers grown in the shady and sunny parts of the garden, at the 5% significance level.

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## What to do if problems arise

If you need to correspond with The Open University about difficulties with submitting your assignments, then keep copies of all correspondence and obtain proofs of posting.

**What should I do if I have prolonged and serious problems in the final two to three weeks before the cut-off dates that will prevent my being able to do the final assignments in time?**

Apply for deferral of your final assignments to the next presentation of the course. This will be granted only if your problems are very serious, such as prolonged illness, or the death or serious illness of a close relative. It will not normally be granted if your problems begin only in the last week before the cut-off date, since you are expected to start work on the final assignments at least two to three weeks before their cut-off dates.

If permission for deferral is granted, your course result letter, which you should receive by mid-December, will state 'pending' instead of pass or fail. You should also receive the TMA 04 and CMA 42 for the next presentation of the course, together with instructions for submitting them, within two weeks of receiving your course result letter. The cut-off dates are in early June.

To apply for deferred submission, write as soon as possible to:

Head of Assessment Credit and Awards
Assessment Policy Office
The Open University
PO Box 83
Walton Hall
Milton Keynes
MK7 6BF

Your application should include:

- the details of your circumstances;
- the timescale in which they have affected or are likely to affect your work;
- documentary evidence supporting your application (for example, a medical certificate).

If you need advice on your application, contact the Learner Support Team at your Regional Centre.

Your application will be acknowledged to confirm receipt.

If possible, you should apply for deferral well before the cut-off date for TMA 04, and continue to work towards your assignments while you wait for a decision on your application, because if deferral is not granted then you will need to submit the assignments in this Assignment Booklet on time in order to pass the course.

If permission for deferral is not granted, then submit whatever you have done by the cut-off date, and also send in a special circumstances (PT39) form (see the next page).

**What should I do if my study is disrupted in the last few days before I need to submit TMA 04 or CMA 42?**

Submit what you have done, and send in a special circumstances (PT39) form (see the next page).

### What should I do if something has adversely affected my performance on the course?

Submit a special circumstances (PT39) form. The Examination and Assessment Board can give limited weight to information about special circumstances that have had a serious adverse effect on your performance in your assessment. The *Assessment Handbook* (accessible via your StudentHome web page) gives guidance regarding the sort of circumstances that the University considers serious and those that it does not. Members of the Learner Support Team, at your Regional Centre, will also be able to offer advice.

You should submit a special circumstances (PT39) form in the following situations.

- You have applied for deferral of your final assignments and it has not been granted.
- Something happens to disrupt your study in the last few days before the cut-off date of one or both final assignments.
- You have had special circumstances affecting your work on one or more assignments earlier in the course.
- You have a disability which you believe has seriously affected your assessment performance.

Note that any information, for example on a disability, that you have given to the University for other purposes will not normally be brought to the attention of the Examination and Assessment Board. It is important that you submit a PT39 form in such circumstances.

A PT39 form can be downloaded from the University website or obtained from the Learner Support Team at your Regional Centre. It should be completed and returned to the Learner Support Team at your Regional Centre, together with supporting documentary evidence, to arrive *no later than 14 days after the cut-off date for TMA 04*. Your tutor cannot submit this information on your behalf.

Special circumstances information received after the deadline given above will not normally be taken into account unless there is evidence that you were unable to notify the University in time. It is your responsibility to make sure that the information reaches the University by the deadline.

### Will the Examination and Assessment Board change the scores awarded by my tutor?

Not normally, but it may adjust your scores up or down if your tutor's marking was considered to be harsh or lenient relative to the mark schemes. Adjustments normally do not exceed 15%.

### What does it mean if I am given a 'pending' result?

It means that the Examination and Assessment Board has been unable to come to a decision about your course result. There are various possible reasons for this. One possibility is that a TMA score or some other information is missing from your assessment record. If this is the case, your final result will be sent to you as soon as possible. Another possibility is that you have not quite managed to achieve the criteria for a pass, and you have been offered the opportunity to attempt new versions of TMA 04 and CMA 42 in the next presentation of the course. You would be told about this along with your result.

**What should I do if I have not received my result by mid-December?**

Write to the Head of Examinations and Assessment, at the address given in the answer to the final question on this page. Please do not ask your Regional Centre, your tutor or the Faculty about your result. Information about results cannot be given over the telephone or by email.

**What should I do if I have been offered the opportunity to attempt the new versions of TMA 04 and CMA 42 in the next presentation of the course, but I have not received these assignments, and the instructions for submitting them, within two weeks of receiving my course result letter?**

Contact the Assignment Handling Office by telephoning 01908 654330 or emailing to assignments@open.ac.uk.

**What should I do if I wish to query or appeal against my course result?**

First, please read carefully the sections in the *Assessment Handbook* about how course results are awarded, result queries and formal appeals to the University. If you wish to proceed, you should write, within four weeks of receiving your result, to:

Head of Examinations and Assessment
The Examinations Office
The Open University
PO Box 720
Walton Hall
Milton Keynes
MK7 6ZQ

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